

Optimization Problem: $\max f(\mathbf{x}) = \text{Google Ads CTR}$

The objective function is simply:

$$f(\mathbf{x}) = \text{CTR} = \frac{\text{Clicks}}{\text{Impressions}}$$

where $\mathbf{x}$ is the full vector of controllable and contextual variables. This is a constrained nonlinear optimization over a mixed (continuous + discrete + categorical) domain. Below is the complete taxonomy of variables $\mathbf{x}$ and their constraints.

Variable Groups in $\mathbf{x}$

1. Bid & Budget Variables

These directly influence **Ad Rank**, which determines ad position, which is the single strongest driver of CTR.

Variable	Symbol	Type
Max CPC bid (per keyword)	b_k	Continuous
Daily/monthly budget	B	Continuous
Bid adjustment multipliers (device, location, time, audience)	$\delta_d, \delta_l, \delta_t, \delta_a$	Continuous
Target CPA (if automated bidding)	tCPA	Continuous
Bid strategy type	—	Categorical

Constraints:

- $b_k \geq b_{\min} > 0$ — minimum bid floor (platform-enforced)
- $b_k \leq b_{\max}$ — max bid ceiling (optionally set by advertiser)
- $\sum_k b_k \cdot \widehat{\text{clicks}}_k \leq B$ — total spend $\leq$ daily budget (inequality)
- $B > 0$
- $\delta_d, \delta_l, \delta_t, \delta_a \in [-0.90, +0.90]$ — Google's allowed bid adjustment range (−90% to +90%)

2. Keyword Variables

Keyword selection and match type directly control *which* searches trigger impressions and how relevant those impressions are — irrelevant impressions inflate the denominator without producing clicks.

Variable	Symbol	Type
Keyword set	$\mathcal{K}$	Discrete / Set
Match type per keyword	$m_k \in \{\text{broad, phrase, exact}\}$	Categorical
Negative keyword set	$\mathcal{N}$	Discrete / Set
Keyword-to-ad-group assignment	—	Categorical

Constraints:

- $\mathcal{N} \cap \mathcal{K}_{\text{active}} = \emptyset$ — a keyword cannot simultaneously be positive and exact-match negative (equality)
- $|\mathcal{K}| \leq \text{platform limit per ad group } (\leq 20,000 \text{ keywords per account})$
- Each keyword $k \in \mathcal{K}$ must have m_k assigned (completeness equality)
- Broader match types (broad $\supset$ phrase $\supset$ exact) imply set inclusion on matched queries

3. Ad Quality Variables (Quality Score Components)

Quality Score (QS, scale 1–10) aggregates three sub-components and feeds directly into Ad Rank = Bid $\times$ QS. A low CTR below $\sim 3\%$ causes a Quality Score penalty.

3a. Expected CTR (eCTR)

Variable	Symbol	Type
Number of RSA headlines (up to 15)	$h \in [1,15]$	Integer
Number of RSA descriptions (up to 4)	$d \in [1,4]$	Integer
Keyword inclusion in headline	$\mathbb{1}_{\text{kw_in_headline}} \in \{0,1\}$	Binary
Call-to-action strength score	s_{CTA}	Continuous (latent)
Ad copy uniqueness / differentiation	s_{diff}	Continuous (latent)

Historical account-level CTR	CTR_{hist}	Continuous
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Constraints:

- $1 \leq h \leq 15$ (equality at max for best RSA performance)
- $1 \leq d \leq 4$
- Each headline ≤ 30 characters (hard platform equality/upper bound)
- Each description ≤ 90 characters
- $\text{CTR}_{\text{hist}} \geq 0$

3b. Ad Relevance

Variable	Symbol	Type
Semantic similarity: keyword $\leftrightarrow$ headline	$\rho_1 \in [0,1]$	Continuous
Semantic similarity: headline $\leftrightarrow$ landing page	$\rho_2 \in [0,1]$	Continuous
Ad group thematic tightness (single-theme SKAG-style)	$\tau \in [0,1]$	Continuous

Constraints:

- $0 \leq \rho_1, \rho_2, \tau \leq 1$ (normalized relevance scores)
- Higher $\tau \leftrightarrow$ fewer, more semantically clustered keywords per ad group

3c. Landing Page Experience (LPX)

Variable	Symbol	Type
Page load speed (seconds)	T_{load}	Continuous
Mobile friendliness score	$s_{\text{mobile}} \in [0,1]$	Continuous
Landing page content relevance to query	$\rho_{\text{LP}} \in [0,1]$	Continuous
Bounce rate	$r_{\text{bounce}} \in [0,1]$	Continuous

Constraints:

- $T_{\text{load}} > 0$, and Google targets $T_{\text{load}} \leq 3\text{s}$ for above-average LPX (soft threshold)
- $0 \leq s_{\text{mobile}}, \rho_{\text{LP}}, r_{\text{bounce}} \leq 1$

- LPX score $\in \{\text{Below Average, Average, Above Average}\}$ — an ordinal constraint on composite $(T_{\text{load}}, s_{\text{mobile}}, \rho_{\text{LP}}, r_{\text{bounce}})$

4. Ad Format & Asset Variables

Ad assets (extensions) expand visual real estate on the SERP and raise Ad Rank, directly improving CTR.

Variable	Symbol	Type
Number of active sitelink assets	$n_{\text{sl}} \in [0,20]$	Integer
Callout extensions active	$n_{\text{co}} \in [0,20]$	Integer
Structured snippet active	$\mathbb{1}_{\text{ss}} \in \{0,1\}$	Binary
Call extension active	$\mathbb{1}_{\text{call}} \in \{0,1\}$	Binary
Price/promotion extension active	$\mathbb{1}_{\text{promo}} \in \{0,1\}$	Binary
Location extension active	$\mathbb{1}_{\text{loc}} \in \{0,1\}$	Binary

Constraints:

- $n_{\text{sl}} \geq 4$ recommended to be eligible for top-of-page sitelink display
- All binary asset variables $\in \{0,1\}$
- Assets only activate if Ad Rank exceeds a threshold (platform-side inequality): $\text{AdRank} \geq \text{AdRank}_{\text{threshold,assets}}$

5. Targeting Variables

These filter the audience receiving impressions, affecting both *volume* and *relevance* of impressions.

Variable	Symbol	Type
Geographic targeting set	$\mathcal{G}$	Discrete / Set
Device targeting (mobile/desktop/tablet)	$\mathcal{D} \subseteq \{\text{mob, desk, tab}\}$	Set
Ad schedule (hours $\times$ days matrix)	$S \in \{0,1\}^{24 \times 7}$	Binary matrix
Audience segment targeting	$\mathcal{A}$	Discrete / Set

Language targeting	$\mathcal{L}$	Discrete / Set
Search partner network inclusion	$\mathbb{1}_{\text{partners}} \in \{0,1\}$	Binary

Constraints:

- $\mathcal{G} \neq \emptyset, \mathcal{D} \neq \emptyset$ — at least one geo and device must be targeted
- $S \neq \mathbf{0}$ — at least one time slot active (campaign must be able to run)
- Device bid adjustments δ_d only apply to devices in $\mathcal{D}$

6. Competitive / Contextual Variables (Partially Uncontrollable)

These are present in $\mathbf{x}$ but impose **exogenous constraints** — the optimizer cannot directly set them but must model them.

Variable	Description	Controllable?
Competitor Ad Ranks $\{AR_j\}_{j=1}^{N_c}$	Determine your ad's position relative to others	X
Auction-time search query intent	Affects eCTR per query	X
SERP organic competition density	Number of organic results competing for clicks	X
User device, location, time at query	Contextual signals used by Google at auction	X
Seasonality / trend index σ_t	Query volume fluctuations	X

Constraints imposed:

- Ad position $p = \text{rank}(\text{AdRank}; \{AR_j\})$ — your position is an ordinal function of all bidders' ranks (inequality system)
- $\text{AdRank} \geq \text{AdRank}_{\min}$ to show at all (platform threshold inequality)
- $\text{Impressions} \leq \text{SearchVolume}(\mathcal{K}) \cdot \text{ImpressionShare}$ — you cannot receive more impressions than the total auction pool allows

Full Constraint Summary

Equality Constraints

$$\text{CTR} = \frac{\text{Clicks}}{\text{Impressions}} \text{ (definition)}$$

$$\text{AdRank} = b_k \cdot \text{QS}(e\text{CTR}, \rho_1, \rho_2, \tau, \rho_{\text{LP}}) \text{ (auction formula)}$$

$$\text{QS} = 1 + \text{LPE_pts} + \text{AR_pts} + \text{eCTR_pts} \in [1, 10]$$

$$m_k \in \{\text{broad, phrase, exact}\} \forall k \in \mathcal{K} \text{ (match type assignment)}$$

$$\mathcal{N} \cap \mathcal{K}_{\text{exact}} = \emptyset \text{ (no positive/negative conflict)}$$

Inequality Constraints

$$b_{\min} \leq b_k \leq b_{\max} \forall k$$

$$\text{AdRank} \geq \text{AdRank}_{\text{threshold}} \text{ (eligibility to serve)}$$

$$\sum_k b_k \cdot \hat{c}_k \leq B \text{ (budget constraint)}$$

$$-0.90 \leq \delta_d, \delta_l, \delta_t, \delta_a \leq 9.00 \text{ (bid adjustment bounds)}$$

$$1 \leq h \leq 15, 1 \leq d \leq 4 \text{ (RSA asset counts)}$$

$$T_{\text{load}} \leq 3\text{s} \text{ (soft LPX threshold)}$$

$$\text{CTR} \geq \text{CTR}_{\text{penalty_threshold}} \approx 3\% \text{ (QS penalty avoidance)}$$

$$0 \leq \rho_1, \rho_2, \rho_{\text{LP}}, r_{\text{bounce}}, s_{\text{mobile}}, \tau \leq 1 \text{ (normalized scores)}$$

Why This is Non-Trivially Hard to Optimize

Several features make $\max f(\mathbf{x})$ a genuinely difficult program:

- **Mixed-integer nonlinearity:** bid variables are continuous, match types and asset flags are discrete/binary, and the CTR function is non-convex in joint bid-quality space.
- **Endogeneity:** CTR is both an *output* (what you maximize) and an *input* (eCTR feeds Quality Score, which affects Ad Rank, which affects position, which affects CTR again — a circular dependency).

- **Partial observability:** competitor Ad Ranks $\{AR_j\}$ are not observable; only proxy signals (Auction Insights, impression share) are available.
- **Non-stationarity:** the objective landscape shifts with seasonality σ_t , competitors changing bids, and Google's algorithm updates — making it a **dynamic** rather than static optimization.
- **Latent quality signals:** Google does not expose the exact weights on eCTR vs. Ad Relevance vs. LPX within QS, nor the full Ad Rank formula beyond its high-level structure.