

Optimization Problem: $\max f(\mathbf{x}) = \text{Visitor} \rightarrow \text{Qualified Lead Rate}$

The objective function is:

$$f(\mathbf{x}) = \frac{\text{Qualified Leads}}{\text{Visitors (post-click)}}$$

This is fundamentally different from the CTR problem: it operates **on your own controllable surface** (the landing page, form, offer, and traffic quality), with almost no uncontrollable external variables. The key tension is a **quality–volume tradeoff** — adding friction reduces total lead volume but increases qualified rate, and vice versa.

Variable Groups in $\mathbf{x}$

1. Traffic Quality Variables

These are inherited from the upstream CTR function — the *type* of visitor who arrives directly affects how many qualify. They enter this function as inputs, not decisions.

Variable	Symbol	Type
Traffic source / channel	$s \in \{\text{paid search, organic, social, direct, ...}\}$	Categorical
Keyword intent score of triggering query	$\iota \in [0,1]$	Continuous
ICP match score (firmographic / demographic fit)	$\phi \in [0,1]$	Continuous
Device type of visitor	$d \in \{\text{mobile, desktop, tablet}\}$	Categorical
Geographic match to target market	$\mathbb{1}_{\text{geo_match}} \in \{0,1\}$	Binary

Constraints:

- $0 \leq \iota, \phi \leq 1$
- These variables are partially controlled upstream (via keyword targeting, audience segments) but are **not directly settable on the landing page** — they impose an **exogenous lower bound on achievable qualified rate**: $f(\mathbf{x}) \leq \phi_{\max}$

2. Landing Page Copy & Offer Variables

What the page says and promises is the primary driver of whether a visitor self-selects to submit.

Variable	Symbol	Type
Headline clarity / specificity score	$c_h \in [0,1]$	Continuous (latent)
Value proposition strength	$c_v \in [0,1]$	Continuous (latent)
Message match: ad copy $\leftrightarrow$ landing page headline	$\mu \in [0,1]$	Continuous
Presence of "who this is NOT for" qualifier block	$\mathbb{1}_{\text{excl}} \in \{0,1\}$	Binary
Price anchor / budget expectation-setting	$\mathbb{1}_{\text{price}} \in \{0,1\}$	Binary
Presence of explainer video	$\mathbb{1}_{\text{video}} \in \{0,1\}$	Binary
Number of distinct CTAs on page	$n_{\text{CTA}} \geq 1$	Integer
Navigation menu present	$\mathbb{1}_{\text{nav}} \in \{0,1\}$	Binary

Constraints:

- $0 \leq c_h, c_v, \mu \leq 1$
- $n_{\text{CTA}} \geq 1$ — at least one CTA required
- $\mathbb{1}_{\text{nav}} = 0$ is strongly preferred: removing navigation doubles conversion vs. pages with full site nav
- Single CTA outperforms multiple by up to 266% — soft constraint: $n_{\text{CTA}} = 1$ is near-optimal
- $\mathbb{1}_{\text{excl}} = 1$ raises qualified lead rate 20–40% by filtering non-ICP visitors before form submission

3. Form Design Variables

Form design is the **most direct mechanical lever** on both volume and quality of leads simultaneously.

Variable	Symbol	Type
Number of form fields	$n_f \geq 1$	Integer
Form field types (text, dropdown, multi-select)	—	Categorical per field
Qualifying fields present (budget range, timeline, company size)	$n_q \leq n_f$	Integer
Form step structure (single-step vs. multi-step)	$\mathbb{1}_{\text{multi}} \in \{0,1\}$	Binary

CTA button text specificity	$s_{\text{CTA}} \in [0,1]$	Continuous (latent)
Inline validation present	$\mathbb{1}_{\text{valid}} \in \{0,1\}$	Binary
Auto-fill / progressive profiling enabled	$\mathbb{1}_{\text{autofill}} \in \{0,1\}$	Binary

Constraints:

- $n_f \geq 1$
- $n_q \leq n_f$ — you cannot have more qualifying fields than total fields (equality upper bound)
- $n_q \geq 1$ — at least one qualifying field is required for the output to be a *qualified* lead (hard equality lower bound — **this is what distinguishes this function from a generic lead-gen function**)
- Volume-quality tradeoff: $\frac{\partial \text{lead_volume}}{\partial n_f} < 0$ while $\frac{\partial \text{qualified_rate}}{\partial n_q} > 0$ — these constraints pull in opposite directions
- Empirically: $n_f = 3$ maximizes raw volume; $n_f \in [4,6]$ with qualifying fields balances volume × quality
- Per HubSpot data on 40,000+ landing pages: each field above 3 reduces completion rate by ~22–44% cumulatively

4. Trust & Social Proof Variables

Trust signals reduce hesitation at the decision moment (adjacent to the CTA) and disproportionately affect *serious* buyers over tire-kickers.

Variable	Symbol	Type
Number of testimonials present	$n_{\text{test}} \geq 0$	Integer
Testimonials include quantified outcomes	$\mathbb{1}_{\text{quant_test}} \in \{0,1\}$	Binary
Client logos present	$\mathbb{1}_{\text{logos}} \in \{0,1\}$	Binary
Security / privacy badge near form	$\mathbb{1}_{\text{sec}} \in \{0,1\}$	Binary
Review count / star rating visible	$\mathbb{1}_{\text{stars}} \in \{0,1\}$	Binary
Case study link present	$\mathbb{1}_{\text{case}} \in \{0,1\}$	Binary
Trust signal placement: above form	$\mathbb{1}_{\text{pos_above}} \in \{0,1\}$	Binary

Constraints:

- All binary variables $\in \{0,1\}$
- $\mathbb{1}_{\text{pos_above}} = 1$ is required for trust signals to affect pre-submission decisions; placement below-fold = effectively zero marginal contribution
- Trust signals collectively increase conversion 34–42%; security badge near form alone can add up to 42%
- $\mathbb{1}_{\text{quant_test}} = 1$ filters for quality: outcome-specific proof attracts buyers, generic praise does not

5. Page Performance Variables

Page speed determines whether a visitor even *engages* with the conversion elements above.

Variable	Symbol	Type
Largest Contentful Paint (LCP)	T_{LCP} (seconds)	Continuous
Interaction to Next Paint (INP)	T_{INP} (ms)	Continuous
Cumulative Layout Shift (CLS)	CLS (unitless)	Continuous
Mobile-first layout	$\mathbb{1}_{\text{mobile}} \in \{0,1\}$	Binary

Constraints:

- $T_{\text{LCP}} \leq 2.5\text{s}$ — Google Core Web Vitals "Good" threshold
- $T_{\text{INP}} \leq 200\text{ms}$ — Core Web Vitals threshold
- $\text{CLS} \leq 0.1$ — Core Web Vitals threshold
- Hard operational floor: $T_{\text{LCP}} > 0$
- At $T_{\text{LCP}} \geq 5\text{s}$, conversion rate degrades to $\sim 1/3$ of the 1-second baseline — effectively a **knockout constraint**: if violated, all other variables become irrelevant
- 53% of mobile visitors abandon pages loading > 3 seconds

6. Qualification Threshold Variables

These define what "qualified" actually means — they are the **output filter** of the function, not inputs.

Variable / Framework	Criteria	Type
BANT: Budget confirmed	$\mathbb{1}_{\text{budget}} \in \{0,1\}$	Binary
BANT: Authority (decision-maker role)	$\mathbb{1}_{\text{auth}} \in \{0,1\}$	Binary
BANT: Need stated	$\mathbb{1}_{\text{need}} \in \{0,1\}$	Binary
BANT: Timing within window	$\mathbb{1}_{\text{timing}} \in \{0,1\}$	Binary
Lead score threshold	$LS \geq \theta_{\text{SQL}}$	Continuous $\geq$ threshold
ICP firmographic match	$\phi \geq \phi_{\text{min}}$	Continuous $\geq$ threshold

Constraints (defining the output):

- A lead is only "qualified" if it clears a composite threshold — a key equality/inequality that defines the domain of the *numerator*:

$$\text{Qualified} = \mathbb{1}[LS(\mathbb{1}_{\text{budget}}, \mathbb{1}_{\text{auth}}, \mathbb{1}_{\text{need}}, \mathbb{1}_{\text{timing}}, \phi) \geq \theta_{\text{SQL}}]$$

- θ_{SQL} is a business decision variable — **raising it reduces numerator count but improves lead quality** (a direct constraint on the objective's definition)
- Minimum: $\theta_{\text{SQL}} > 0$ — without a threshold, the function collapses to generic visitor-to-lead, not visitor-to-*qualified*-lead

Full Constraint Summary

Equality Constraints

$$f(\mathbf{x}) = \frac{\sum_i \mathbb{1}[LS_i \geq \theta_{\text{SQL}}]}{\text{Visitors}} \quad (\text{definition of objective})$$

$$n_q \leq n_f \quad (\text{qualifying fields cannot exceed total fields})$$

$$n_q \geq 1 \quad (\text{at least one qualifying field required for qualified output})$$

Inequality Constraints

$$T_{\text{LCP}} \leq 2.5\text{s}, T_{\text{INP}} \leq 200\text{ms}, \text{CLS} \leq 0.1 \text{ (Core Web Vitals — knockout constraints)}[]$$

$$0 \leq \iota, \phi, \mu, c_h, c_v \leq 1 \text{ (normalized score bounds)}$$

$$\theta_{\text{SQL}} > 0 \text{ (qualification threshold must be positive)}$$

$$n_f \geq 1, n_{\text{CTA}} \geq 1$$

$$n_f \in [4,6] \text{ (soft empirical bound: quality-volume balance)}[][]$$

$$\phi \geq \phi_{\min} \text{ (minimum ICP match score to count as qualified)}[]$$

The Core Tension: Volume vs. Quality

Unlike the CTR function, this objective has a **built-in adversarial tradeoff**: increasing qualification stringency (raising θ_{SQL} , adding qualifying fields n_q , including the exclusion block $\mathbb{1}_{\text{excl}}$) simultaneously *decreases* the denominator contribution to f through lower raw conversion volume, but *increases* the numerator fraction that passes the qualified threshold. The optimizer must therefore treat this as a **Pareto-frontier problem** unless a single weighted objective or minimum qualified-lead volume constraint is added:

$$\text{Qualified Leads} \geq L_{\min} \text{ (minimum volume constraint — business requirement)}$$

Without this floor, the trivially optimal solution is to make the form so demanding that only one perfect lead per month submits — 100% qualified rate, near-zero business value.